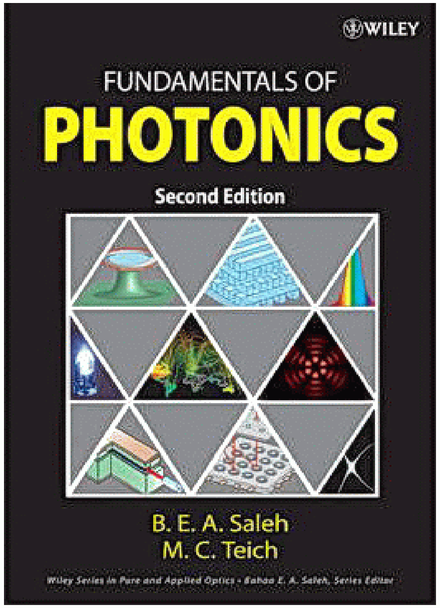
WARSAW UNIVERSITY OF TECHNOLOGY
DEVELOPMENT PROGRAMME

Quantum Electronics
Lecture 1


WILEY

FUNDAMENTALS OF
PHOTONICS
Second Edition

B. E. A. Saleh
M. C. Teich

Wiley Series in Pure and Applied Optics • Bahaa E. A. Saleh, Series Editor

Introduction to Quantum Electronics

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HUMAN CAPITAL
NATIONAL COHESION STRATEGY

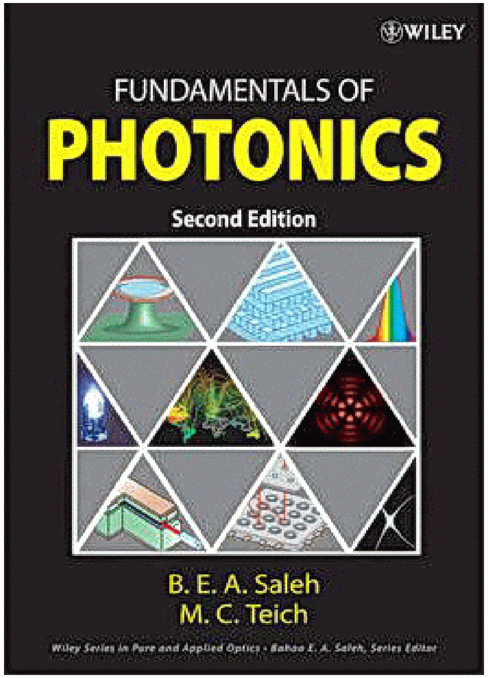
EUROPEAN UNION
EUROPEAN
SOCIAL FUND

Lectures co-financed by the European Union in scope of the European Social Fund

Contents

- ***Course formalia***
- ***Introduction to Quantum Electronics***
- ***Résumé of Electromagnetic theory***
- ***Optical coherence***

Course literature



***Very helpful* for the major part of the course topics**

Book content:

http://www.wiley-vch.de/publish/en/books/ISBN0-471-35832-0/

Other materials* for the topics not covered by the course book can be found in *http://www.ict.kth.se/courses/IO2655/index.htm?links.html

*Especially note **two online books** under: Electromagnetics, Optics and Photonic crystals*

Credits and requirements

2 ECTS credits

↑

To get the course approved you need to pass the exam

Examination – May 20

The written, close book exam will consist of questions or simple problems based on the lecture material

Formulas if needed will be provided with the exam sheet

50% right answers are required to pass

What is Quantum Electronics, Photonics, and Optics?

***Quantum Electronics:* “A loosely defined field concerned with the interaction of radiation and matter, particularly interactions involving quantum energy levels and resonance phenomena”**

***Quantum electronics* is approximately synonymous with**

***Photonics (Optical Electronics)* – the science of generating, controlling, and detecting photons - optical equivalent of electronics**

***Both include*:**
emission, transmission, amplification, detection, modulation, and switching of light

Classical optics is a sub-set of photonics. It covers part of light controlling
Modern optics is commonly categorized as photonics

http://www.answers.com/topic/quantum-electronics

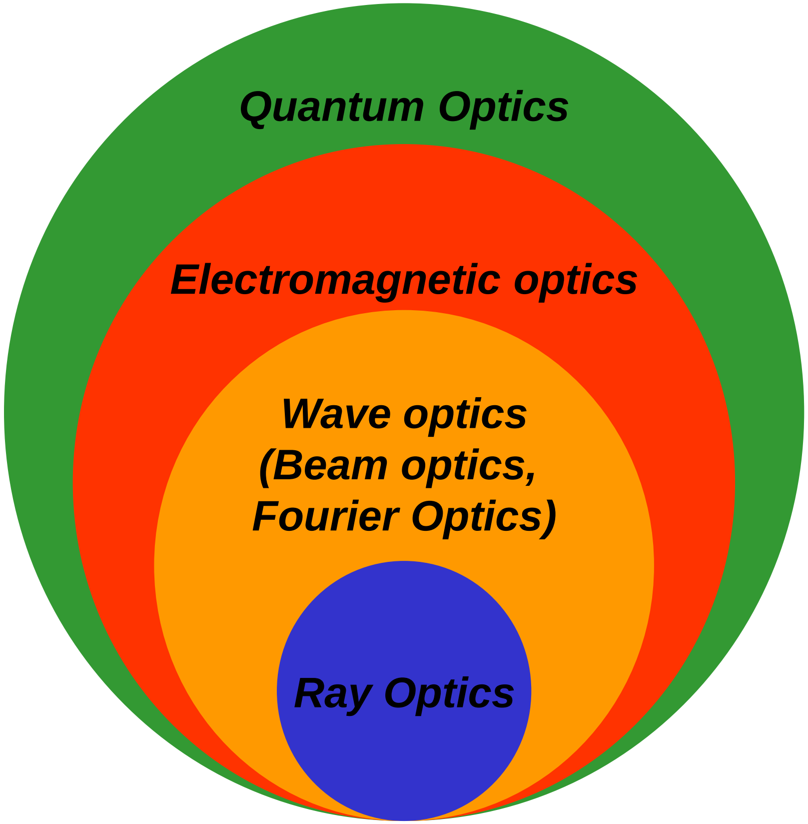
Descriptions of Optical Phenomena

The theory of quantum optics
provides an explanation of virtually all optical phenomena

The electromagnetic theory *of light (electromagnetic optics) provides the most complete treatment of light within the confines of classical optics*

Wave optics *is a scalar approximation of electromagnetic optics*

Ray optics *is the limit of wave optics when the wavelength is very short compared with structures*



Good link shortly reviewing Electromagnetics fundamentals:
http://www.dur.ac.uk/g.h.cross/notes_b.pdf

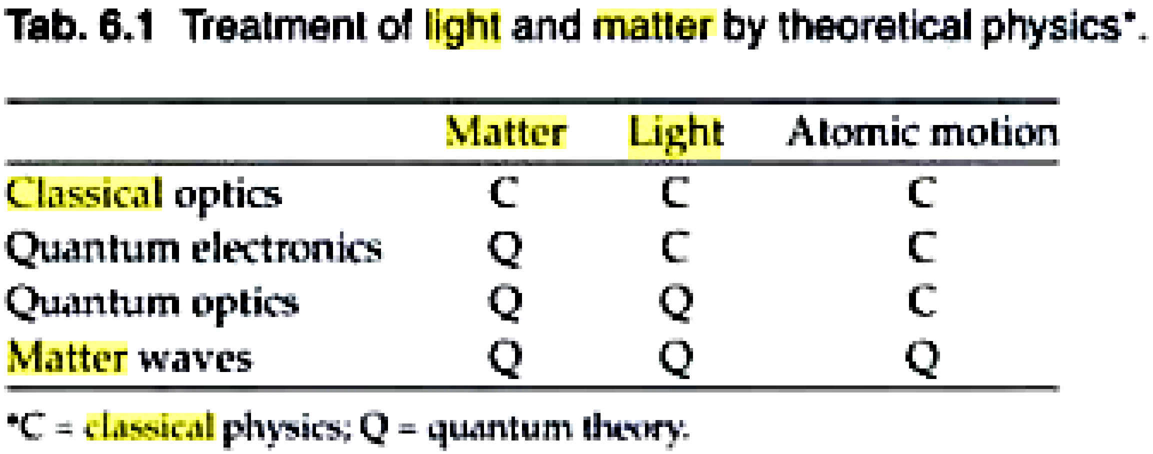
Light Matter Interaction - levels of treatment

Classical: Lorentz dipole oscillator

Semiclassical: atoms quantized, light classical

2nd quantization: light and atoms quantized

Full Quantum Electrodynamical (QED)



Tab. 6.1 Treatment of light and matter by theoretical physics*.

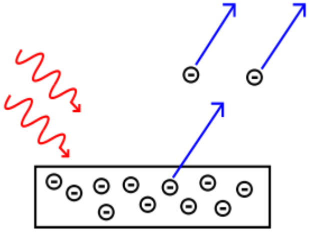
	Matter	Light	Atomic motion
Classical optics	C	C	C
Quantum electronics	Q	C	C
Quantum optics	Q	Q	C
Matter waves	Q	Q	Q

*C = classical physics; Q = quantum theory.

History of light and matter interactions

Heinrich Hertz
1857 –1894

Heinrich Hertz 1887 -
discovered photoelectric effect



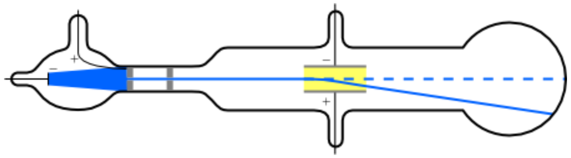
Lorentz 1892 - Classical electron theory
- Nobel prize 1902 (with Zeeman)

Hendrik Lorentz
1853-1928

Described the electromagnetic force acting on a charged particle
Atom - nucleus connected to electrons by a “spring”

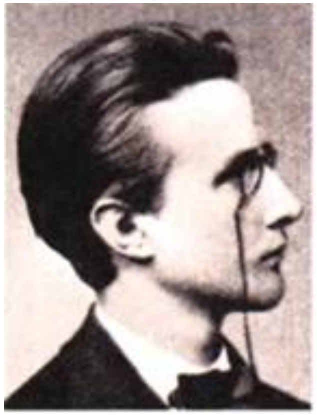
Joseph Thomson
1856-1940

Thomson 1897 – ”Discovered” electrons
- Nobel prize 1906



Found that the cathode rays are deflected by an electric field and concluded that they were **negatively charged particles**

History of light and atom quantization (1)



Max Planck
1858-1947

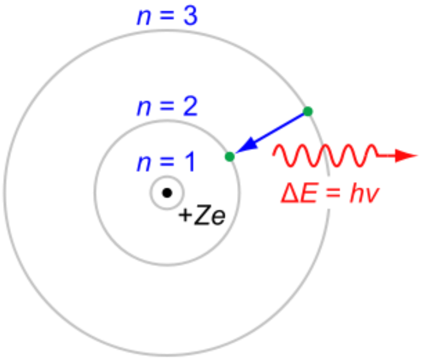
Planck 1900 – **Quantized energy of atomic radiators**, explains black body radiation - Nobel prize 1918

Einstein 1905 - **Light quanta postulate** explains photoelectric effect - Nobel prize 1921

Albert Einstein
1879-1955

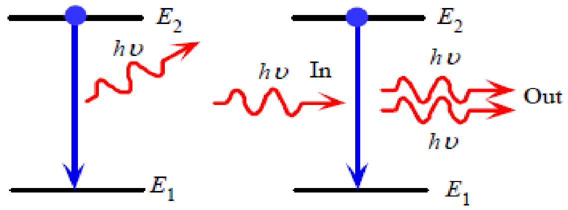
Niels Bhor
1885-1962

Bohr 1913 – **Quantized atom model**, explains spectral lines of hydrogen atom - Nobel prize 1922

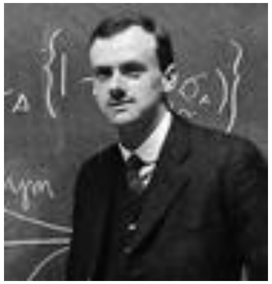


History of light and atom quantization (2)

Einstein 1917 **–** **Quantified spont. emission, discoverd stimulated emission**



Dirac 1927 - **Light-field quantization, relativistic description of electron** - *Nobel prize 1933 (with Schrödinger)*



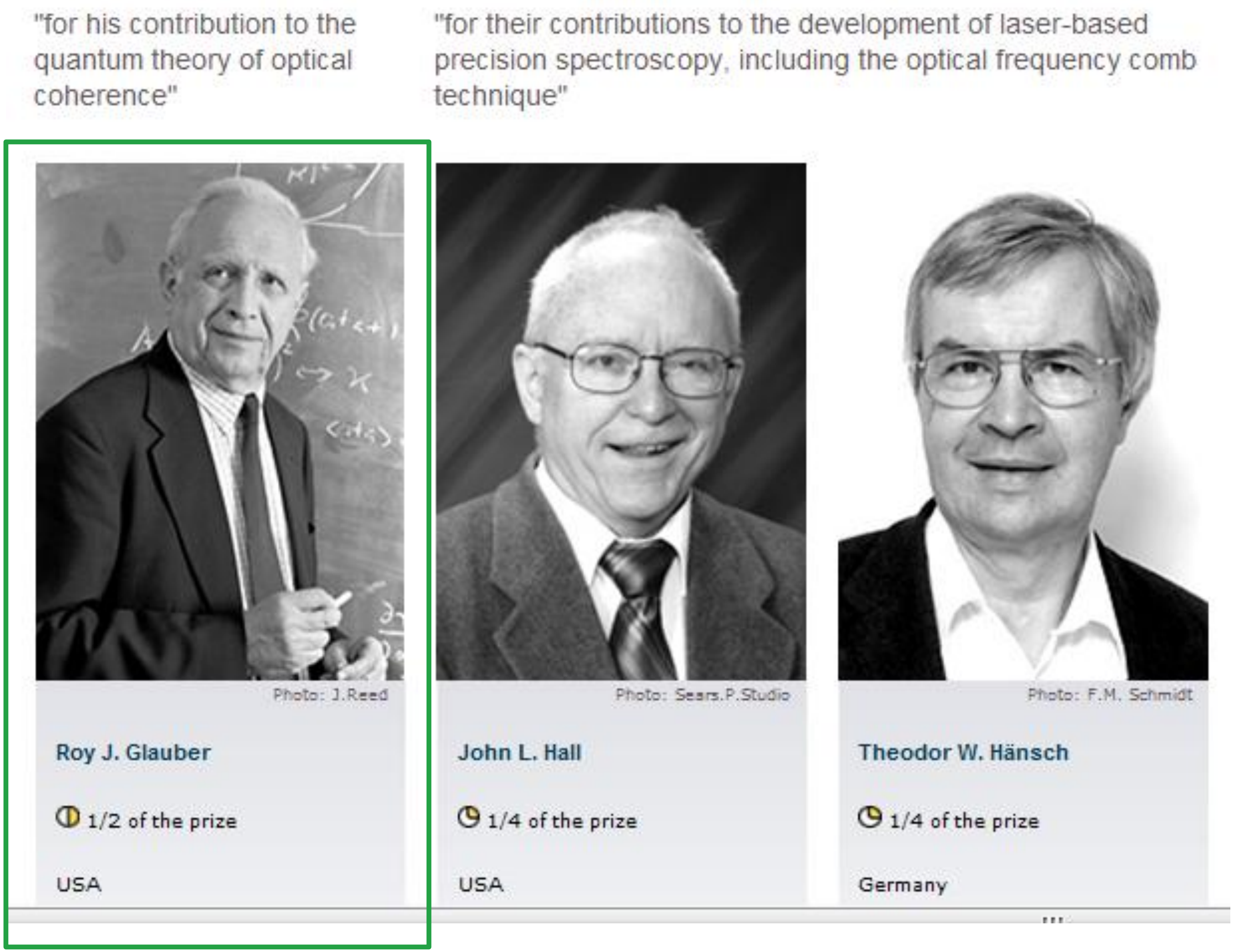
Paul Dirac
1902-1984

Richard Feynman
1918–1988

Feynman, Dyson, Schwinger, Tonagawa 1940s - **Quantum electrodynamics** - *Nobel prize 1965*

Glauber 1963 **– Fomulation of quantum theory to describe the detection process**
- *Nobel prize 2005*

Nobel Prize in Physics 2005 for breakthroughs in modern optics



"for his contribution to the quantum theory of optical coherence"

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

Photo: J.Reed

Roy J. Glauber

◑ 1/2 of the prize

USA

Photo: Sears.P.Studio

John L. Hall

◔ 1/4 of the prize

USA

Photo: F.M. Schmidt

Theodor W. Hänsch

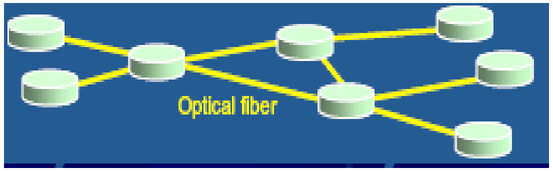
◔ 1/4 of the prize

Germany

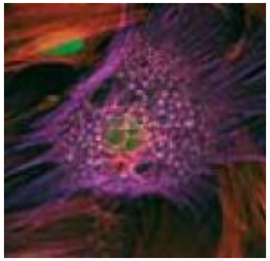
Showed how the quantum theory has to be formulated to describe the detection process

Applications of Optoelectronics

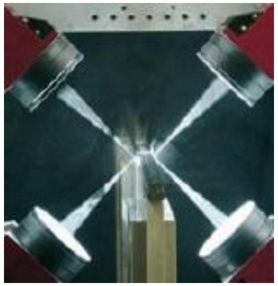
← ***Information, Communication*** →



Imaging →



← ***Lighting and Displays***



← ***Manufacturing and Quality***

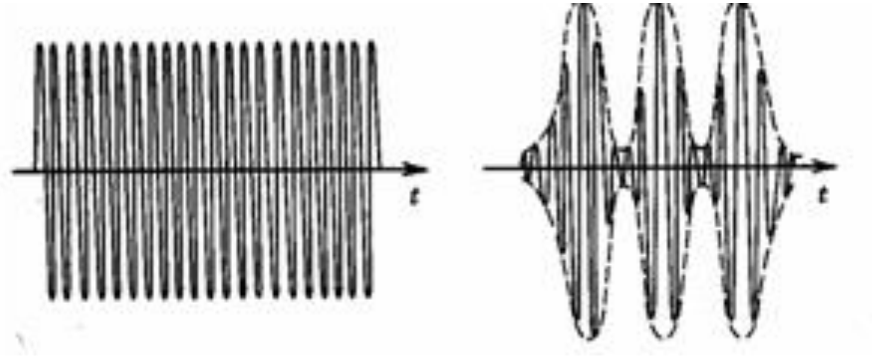
Life Science and Health Care →

Safety and Security →

http://www.photonics21.org/

High-capacity communications networks

Information-carrying capacity of light
>> (10,000 times) than at radio frequencies



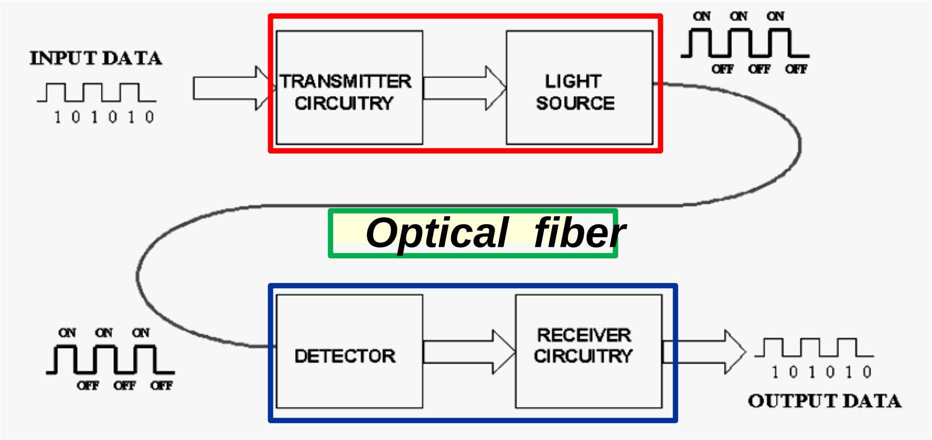
➡ *High-speed / capacity communications networks*

Think what this for instance means for the speed of your internet !!!

Or for the quality of image transfer

Basic fiber optic system

***Transmitter* - converts an electrical signal into a light signal**
***Optical fiber* - carries the light**
***Receiver* - accepts the light signal and converts it back into an electrical signal**



Relies on: ***Low loss fiber transmission***
Light generators and detectors
Opto-electronic interface / integration

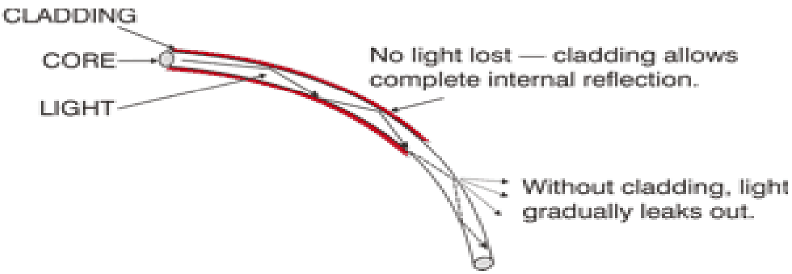
Light guiding by internal reflection

Daniel Colladon's Experiment ("light jet") 1841

1950's - first practical all-glass fiber developed and applied for image transmission (fiberscope)

by Brien O'Brien at Narinder Kapany (USA)

He coined the term "fiber optics" - 1956



Cladding introduced by van Heel protected core surface from contamination and reduced losses

Still in 1960 very high transmission loss - 10 000 dB/km!

Towards optical communication

Breakthroughs:

1966 High fiber loss attributed to impurities (not silica glass)

Losses < 20 dB/km possible, Long-distance communications over single-mode fiber proposed (Standard Telecommunications Laboratories, UK - Kao and Hockham) *Research initially supervised by Antoni E. Karbowiak*

1970 First fiber with loss < 20 dB/km at 633 nm (helium-neon) demonstrated (Corning - Maurer, Keck, Schultz)

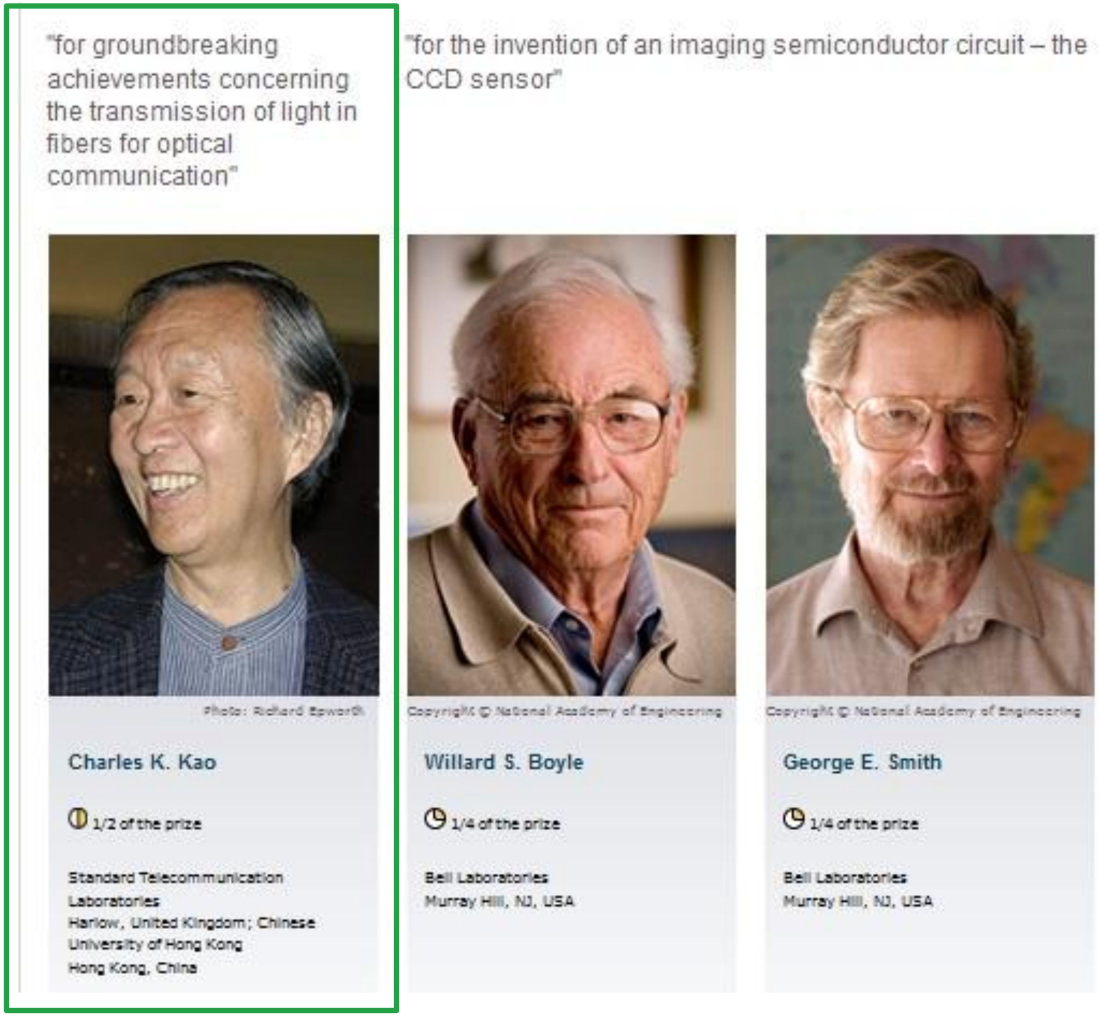
1970 First continuous-wave room-temperature semiconductor lasers demonstrated (Ioffe Physical Institute - Alferov's group, Bell Labs – Panish and Hayashi)

1978 0.2 dB/km loss in single-mode fiber at 1.55 um (NTT) !!!

1987 First erbium-doped fiber amplifier for 1.55 um demonstrated

(Southampton University – Payne et al, AT&T Bell Laboratories – Desurvire et al)

2009 Nobel Prize in Physics for the masters of light



"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"

"for the invention of an imaging semiconductor circuit – the CCD sensor"

Photo: Richard Epworth

Charles K. Kao

1/2 of the prize

Standard Telecommunication Laboratories
Harlow, United Kingdom; Chinese University of Hong Kong
Hong Kong, China

Copyright © National Academy of Engineering

Willard S. Boyle

1/4 of the prize

Bell Laboratories
Murray Hill, NJ, USA

Copyright © National Academy of Engineering

George E. Smith

1/4 of the prize

Bell Laboratories
Murray Hill, NJ, USA

http://nobelprize.org/nobel_prizes/physics/laureates/2009/info_publ_phy_09_en.pdf

The physics behind IT - Nobel price in Physics 2000

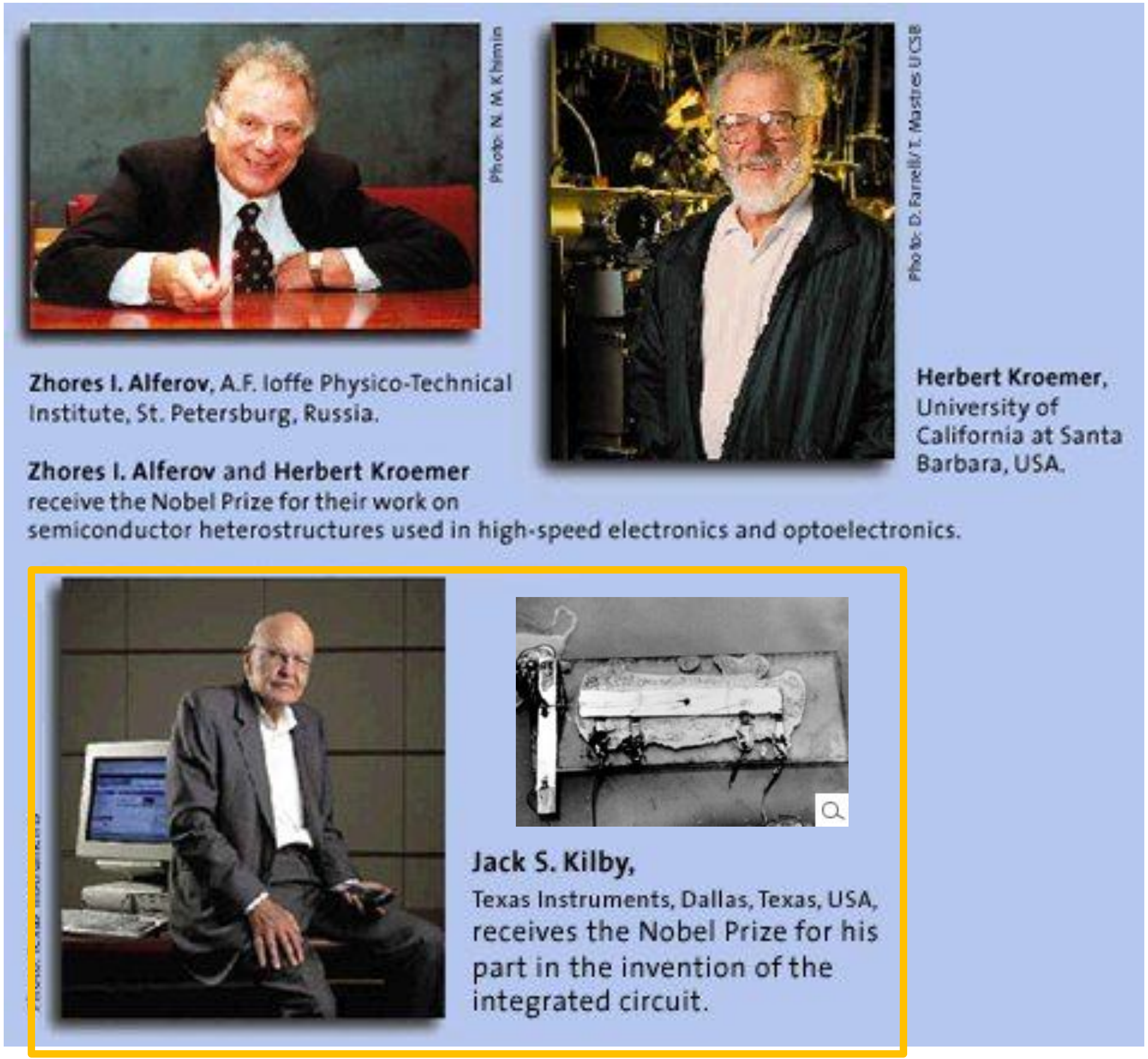


Photo: N. M. Khimin

Photo: D. Farrell/ T. Mastres UCSB

Zhores I. Alferov, A.F. Ioffe Physico-Technical Institute, St. Petersburg, Russia.

Herbert Kroemer, University of California at Santa Barbara, USA.

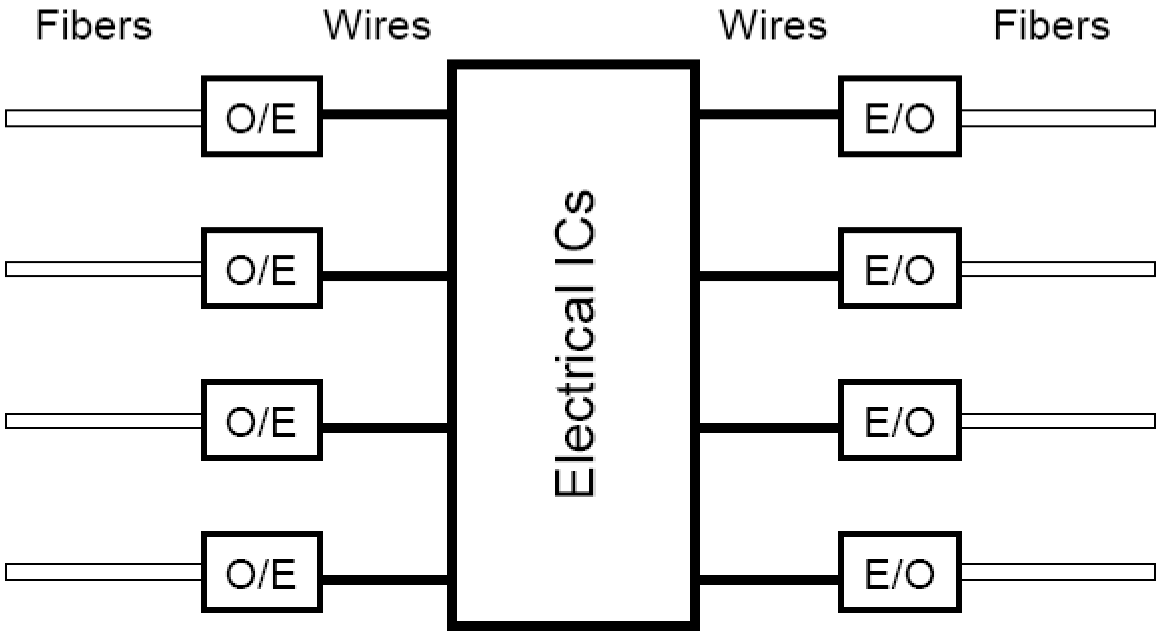
Zhores I. Alferov and Herbert Kroemer receive the Nobel Prize for their work on semiconductor heterostructures used in high-speed electronics and optoelectronics.

Jack S. Kilby,

Texas Instruments, Dallas, Texas, USA, receives the Nobel Prize for his part in the invention of the integrated circuit.

Invention of the electrical integrated circuit in 1958

Signal processing in electronic domain

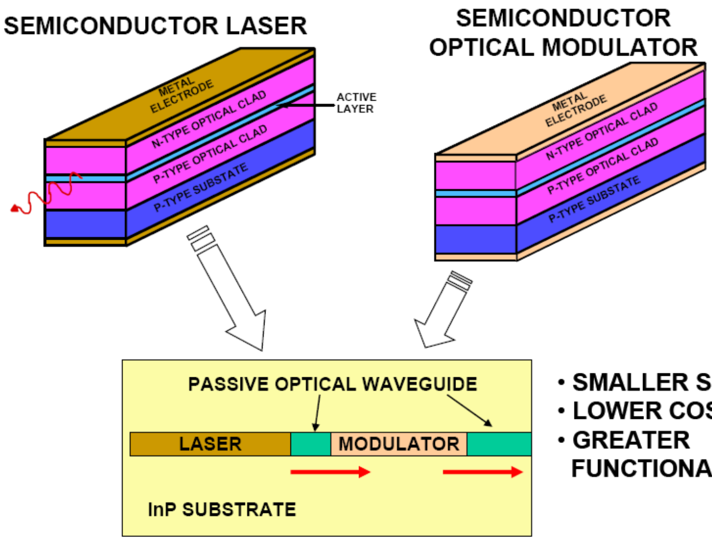


Switching, Add-drop Multiplexing
Wavelength Conversion, Signal Reshaping…

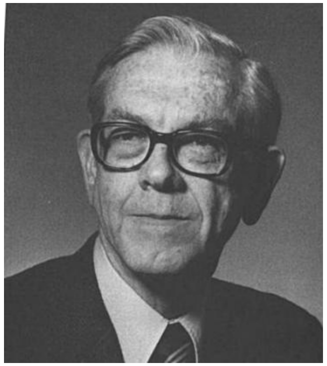
O/E, E/O converters are bit rate and wavelength dependent
Bandwidth narrower than the optical one

Integrated optics / photonics

In 1960 Miller proposed integration of several optical components on one semiconductor chip and coined the term "integrated optics" - analogy to electronic integrated circuits



- **SMALLER SIZE**
- **LOWER COST**
- **GREATER FUNCTIONALITY**



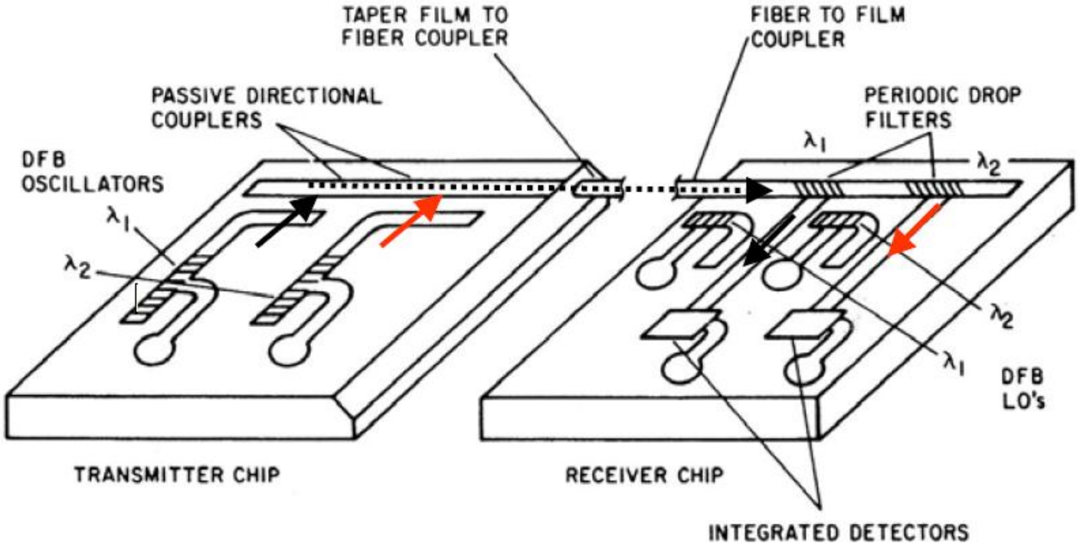
Steward E. Miller
1918 - 1990

The uses of photons instead of electrons would eliminate O/E converters - hence make the chip bit rate and wavelength transparent

AIM:

Light sources, modulators, switches, filters, splitters, waveguides, and detectors on a single integrated platform

Signal processing in optical domain



Complex photonic Integrated circuit (PIC) is still a "holy grail" ...

Drawbacks:

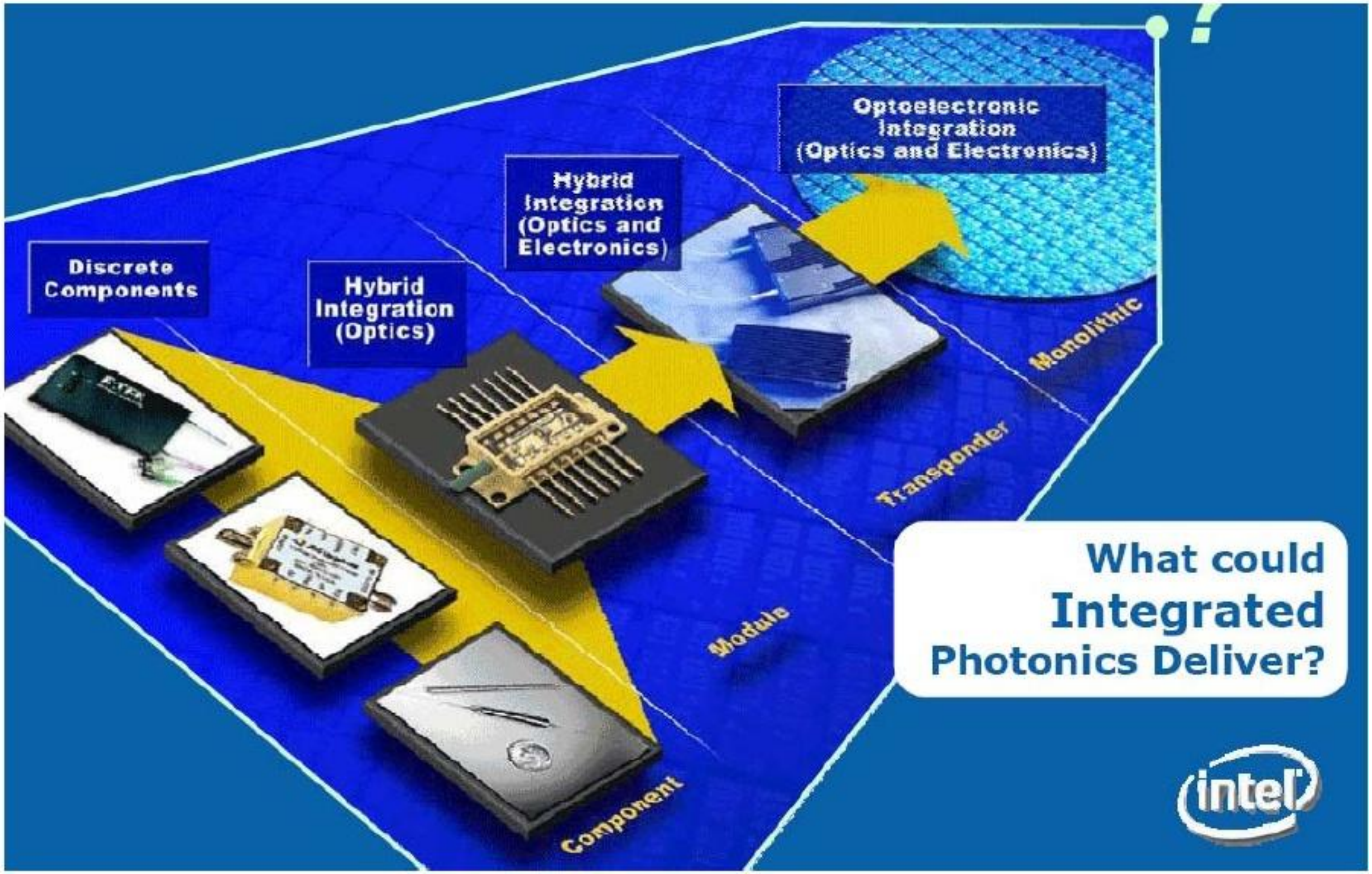
Big!! – at least 1000 times larger than its electronic counterpart
High cost of developing new fabrication technology

"holy grail" - any ultimate, but elusive, goal pursued as in a quest

Święty Graal nadal pozostaje tajemnicą a legendy mówią, że aby doznać oświecenia należy poznać tajemnicę Graala

The ongoing efforts to miniaturize PIC will be addresed in the next lectures

Optoelectronic Integration



Maxwell equations

Ampère's circuital law (with Maxwell's correction)

$$\nabla \times H = \boxed{\frac{\partial D}{\partial t}} + J \qquad \nabla \bullet D = \rho$$

Gauss's law

Faraday's law of induction

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \bullet B = 0$$

Gauss's law for magnetism

$D = \varepsilon E, \quad B = \mu H$

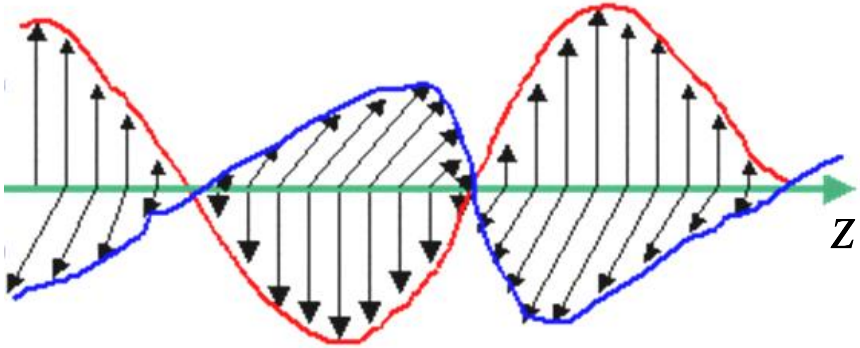
Maxwell's correction: *electric field changing in time generates magnetic field*
Faraday's low: *a changing magnetic field induces an electric field*

Self-sustaining "electromagnetic waves" can travel through empty space

http://www.plasma.uu.se/CED/Book/ - online book on **"Electromagnetic Field Theory" by Bo Thidé**
For Si units see e.g. http://en.wikipedia.org/wiki/Maxwell's_equations

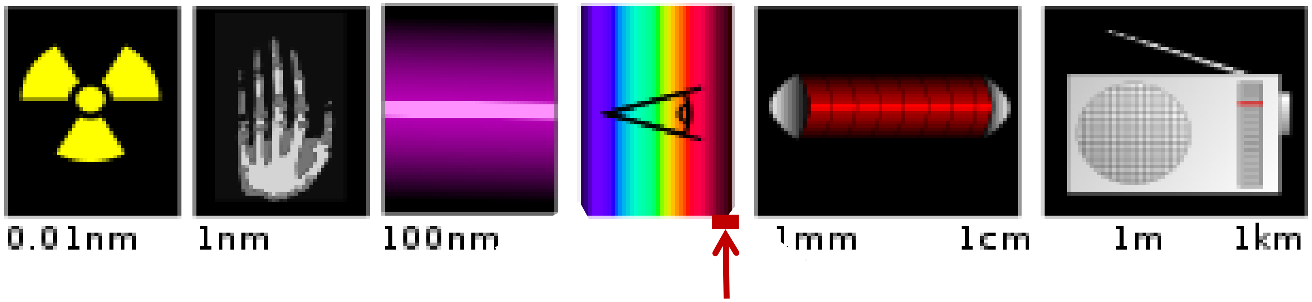
Electromagnetic Wave

Maxwell's hypothesis 1864: light is an electromagnetic wave



Electric and magnetic fields are oscillating perpendiculary to each other and to the direction of propagation

Ranges of Electromagnetic Waves



Wikipedia

Optical communications
1260 - 1675 nm
(near & short infrared)

Name	Frequency	Wavelength (λ)	Time for one λ
Extra Low Freq	60 Hz	5000 km (5×10^{6})	17 ms (1.7×10^{-2})
Audio Frequency	10 kHz (1×10^{4})	30 km (3×10^{4})	100 μs (1×10^{-4})
Radio Frequency	222 MHz (2×10^{8})	1.4 m	4.5 ns (4.5×10^{-9})
Microwave	10 GHz (1×10^{10})	30 mm (3×10^{-2})	100 ps (1×10^{-10})
Infrared (Heat)	10 THz (1×10^{13})	30 μm (3×10^{-5})	100 fs (1×10^{-13})
Visible	600 Thz (6×10^{14})	500 nm (5×10^{-7})	1.7 fs (1.7×10^{-15})
Ultraviolet	1×10^{16} Hz	30 nm (3×10^{-8})	.1 fs (1×10^{-16})
X-ray	1×10^{18} Hz	300 pm (3×10^{-10})	1×10^{-18} s
Gamma-ray	1×10^{20} Hz	3 pm (3×10^{-12})	1×10^{-20} s

Wave equation

$$\boxed{\begin{array}{ll} \nabla\times H=\dfrac{\partial D}{\partial t}+J & \nabla\cdot D=\rho \\ \nabla\times E=-\dfrac{\partial B}{\partial t} & \nabla\cdot B=0 \end{array}} \xrightarrow[\rho=0]{J=0} \nabla^2 E-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}=-\nabla\left(\frac{1}{\varepsilon}E\cdot\nabla\varepsilon\right)$$

ε *defines medium properties*

When ***ε*** *is constant, or varies slowly in comparison with the optical wavelength:*

$$\boxed{\nabla^2 E-\mu\varepsilon\frac{\partial^2 E}{\partial t^2}=0}$$

Wave equation

For monochromatic
(single harmonic) fields:

$$\boxed{\nabla^2 E+k^2 E=0}$$

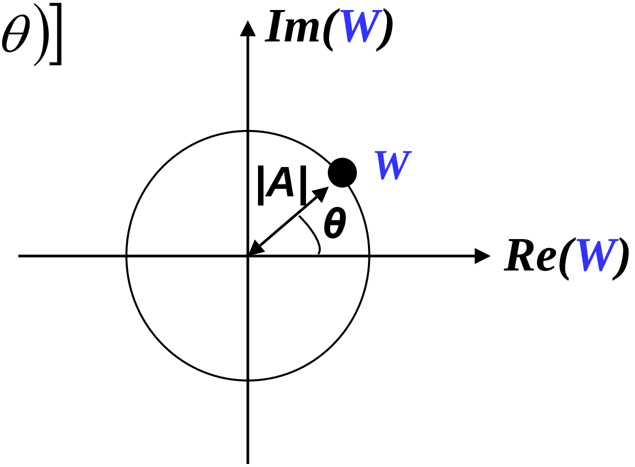
Helmholtz equation

Plane wave:

$$\boxed{\boxed{E=A\cos(kz-\omega t)}} \qquad k=\omega\sqrt{\mu\varepsilon}=\frac{\omega}{c}=\frac{2\pi}{\lambda}$$

Complex number convention

Complex number: $W = Ae^{i\theta} = A[\cos(\theta) + i\sin(\theta)]$

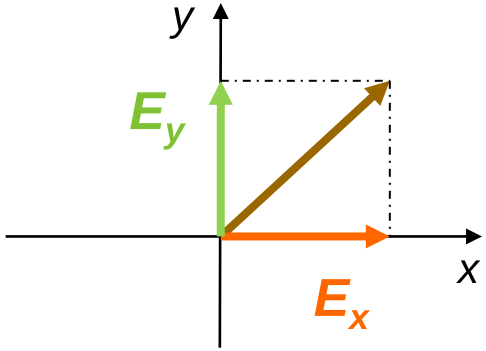


Plane wave: $E = A\cos(kz - \omega t)$

$$E = \mathrm{Re}\left[Ae^{i(kz-\omega t)}\right] \equiv \frac{1}{2}\left[Ae^{i(kz-\omega t)} + \underset{\substack{\uparrow \\ \text{complex conjugate}}}{c.c.}\right]$$

For simplicity ½ Re, or +c.c are commonly omitted: $\boxed{E = Ae^{i(kz-\omega t)}}$

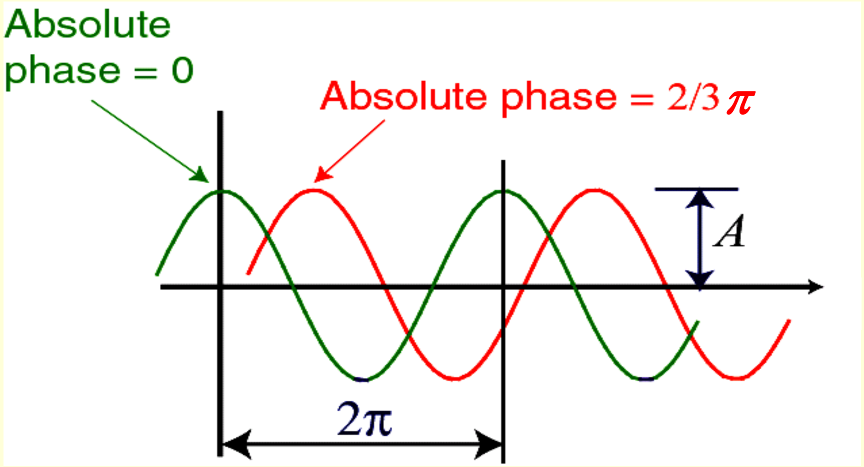
Plane wave – complex amplitude



$$\boxed{\vec{E}(k,t)=\hat{i}_x E_x(k,t)+\hat{i}_y E_y(k,t)}$$

$$E_x = A_x e^{i(kz-\omega t+\varphi_x)}$$

$$E_y = A_y e^{i(kz-\omega t+\varphi_y)}$$



$$\vec{E} = \underbrace{\left[\hat{i}_x A_x e^{i\varphi_x}+\hat{i}_y A_y e^{i\varphi_y}\right]}_{\text{complex amplitude}} e^{i(kz-\omega t)}$$

Light polarization

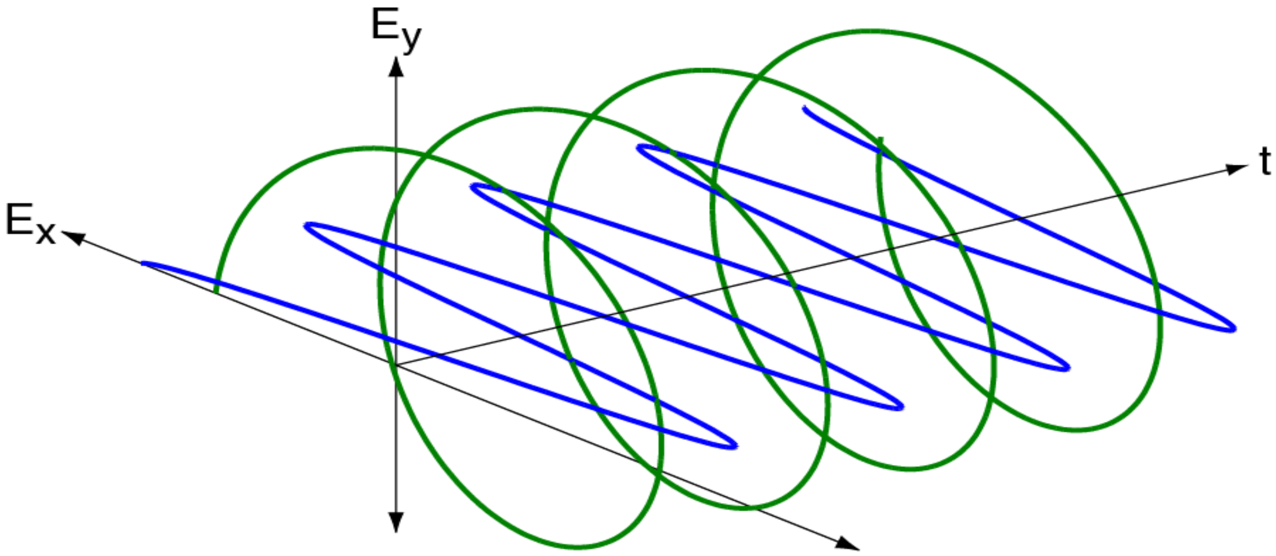
Polarization of a plane wave propagaiting in **z** direction is defined as the curve traced in time in the **xy plane** by the end point of the electric field vector.

LINEAR:

$$\mathbf{E}(t) = \hat{\mathbf{x}}\, E_0 \cos(\omega t)$$

CIRCULAR:

$$\mathbf{E}(t) = \frac{E_0}{\sqrt{2}}\left[\hat{\mathbf{x}} \cos(\omega t) + \hat{\mathbf{y}} \sin(\omega t)\right]$$

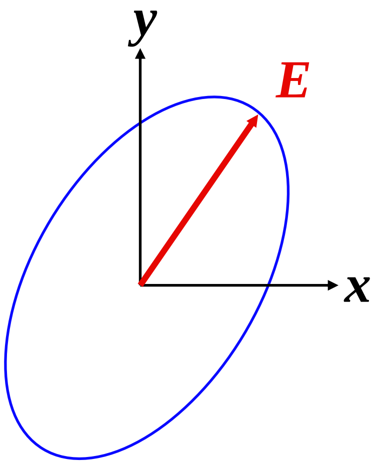


Types of Polarization

In general, light is *elliptically polarized*

$$\left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - \frac{2E_xE_y\cos\delta}{A_xA_y} = \sin^2\delta$$

$$\delta \equiv (\varphi_x - \varphi_y)$$



Special cases:

Zero phase difference (δ= 0) gives oscillation along a line: linear polarization

Equal amplitudes (Ax = Ay) and π/2 phase difference: circular polarization

Birefringence

$$D = \varepsilon \cdot E \quad \textbf{and} \quad n^2 \equiv \varepsilon$$

depends on the direction, so ε is a tensor

uniaxial crystal: $n_x = n_y \equiv n_0 \neq n_z \equiv n_e$ ***Birefringence***

ordinary index
(perpendicular to optic axis z)

extraordinary index
(along optic axis z)

In the pricipial coordinate system off-diagonal elements vanish:

$$D_x = \varepsilon_{11} E_x = n_0^2 E_x \qquad D_y = \varepsilon_{22} E_y = n_0^2 E_y \qquad D_z = \varepsilon_{33} E_z = n_e^2 E_z$$

In general, directions of E and D are different!

Impermeability tensor – Index elipsoid

$E = \varepsilon^{-1} D$ **Define:** $\eta = \frac{1}{\varepsilon}$ ***Impermeability tensor:*** $\boxed{\eta_{ij} = \frac{1}{n_{ij}^2}}$

Symmetric in lossless and optically inactive media : $\boxed{\eta_{ij} = \eta_{ji}}$

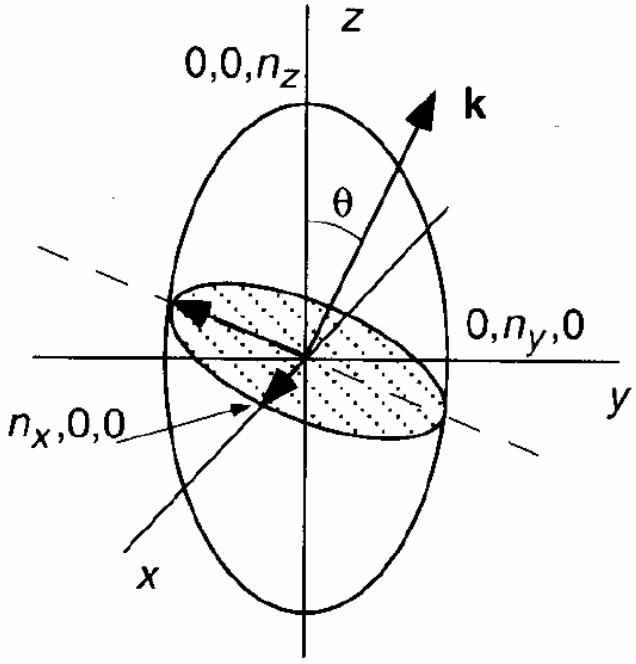
$$\boxed{\eta_{ij} x_i x_j \equiv \sum_{ij} \eta_{ij} x_i x_j = 1}$$

***the index ellipsoid* - convinient geometric representation**

in the principal coordinate system (crystal axes):

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

where: $x_{11} \equiv x, \;\; x_{22} \equiv y, \;\; x_{33} \equiv z$



Ordinary and extraordinary waves

In uniaxial crystals : $\frac{x_1^2}{n_0^2}+\frac{x_2^2}{n_0^2}+\frac{x_3^2}{n_e}=1$

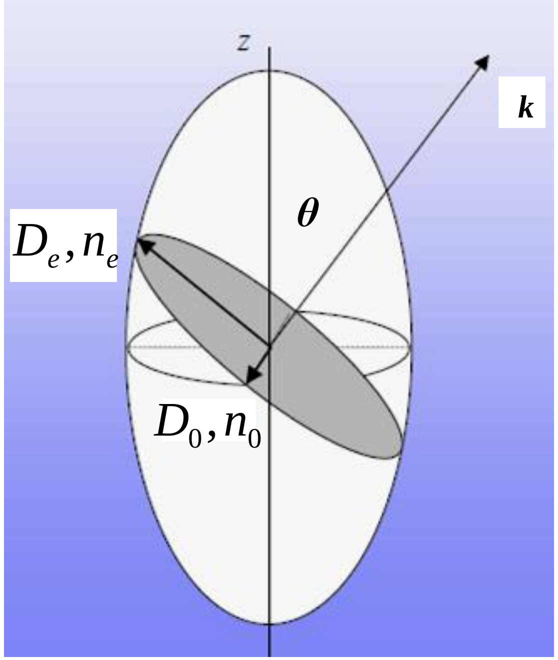
For any propagation direction k there are two allowed waves of two orthogonal polarizations:

Ordinary wave: D_0 ***in the plane perpendicular to z for which*** $n=n_0$

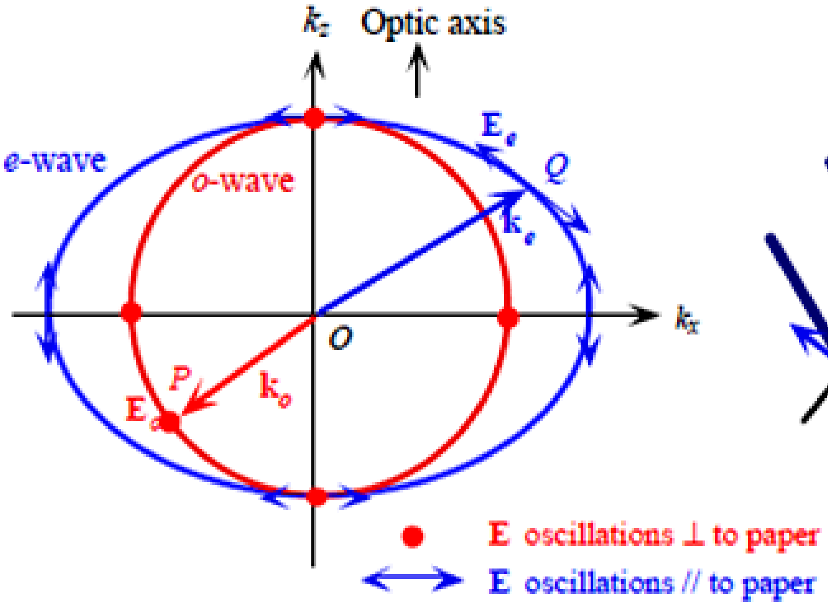
Extraordinary wave: D_e ***perpendicular to*** D_0
n ***depends of the propagation direction:***

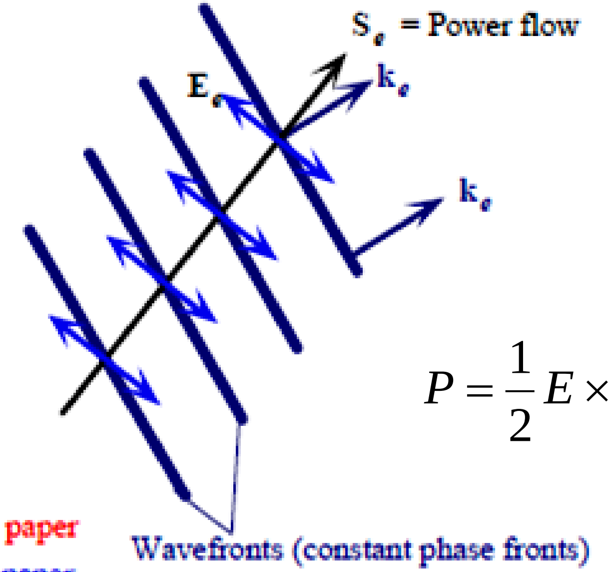
$$\frac{1}{n^2(\theta)}=\frac{\cos^2(\theta)}{n_0^2}+\frac{\sin^2(\theta)}{n_e^2} \quad \longrightarrow \quad n(0°)=n_0 \quad n(90°)=n_e$$

Both D_0 ***and*** D_e ***are perpendicular to k***



Normal surface or wavevector surface

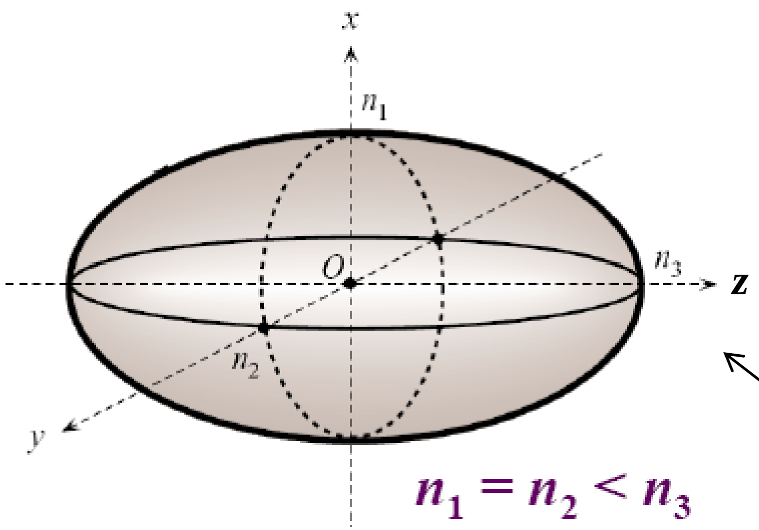




$$P = \frac{1}{2} E \times H^*$$

For extraordinary wave D is perpendicular to k, E to S !

Positive and negative birefringence



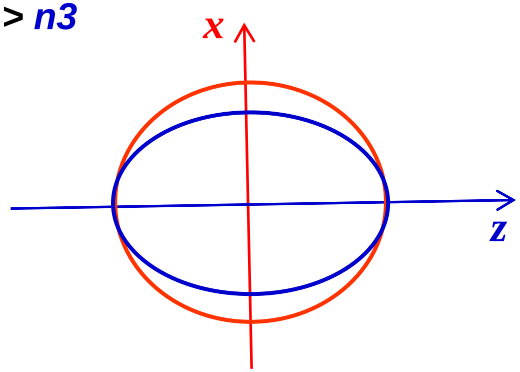
Optic axis

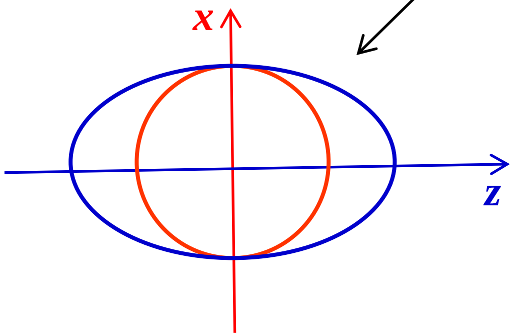
$n_1 = n_2 < n_3$

Positive birefringence

Negative birefringence:

$n1 = n2 > n3$

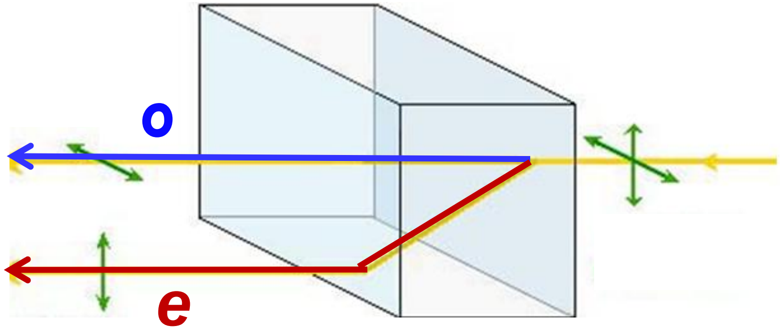


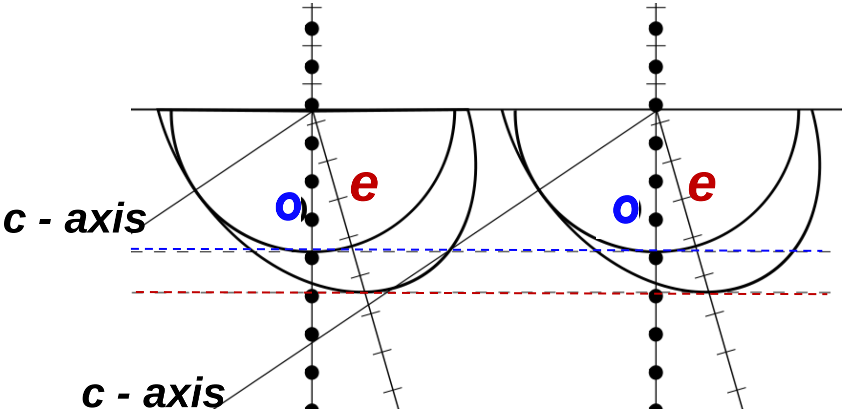


$\frac{c}{n_e} \geq \frac{c}{n_o}$ *o-wave travels slower*

$\frac{c}{n_e} \leq \frac{c}{n_o}$ *e-wave travels slower*

Double refraction





o-ray (ordinary)

Obeys Snell's Law and goes straight

Vibrates ⊥ plane containing ray and c-axis ("optic axis")

e-ray (extraordinary)

Deflected

*Vibrates **in** plane containing ray and c-axis*

Double image:

Group velocity in a medium

$$v_g \equiv d\omega/dk$$

$$\boxed{v_g \equiv \left[dk/d\omega\right]^{-1}}$$

Using $k = \omega\, n(\omega)/c_0$, *calculate:* $dk/d\omega = (n + \omega\, dn/d\omega)/c_0$

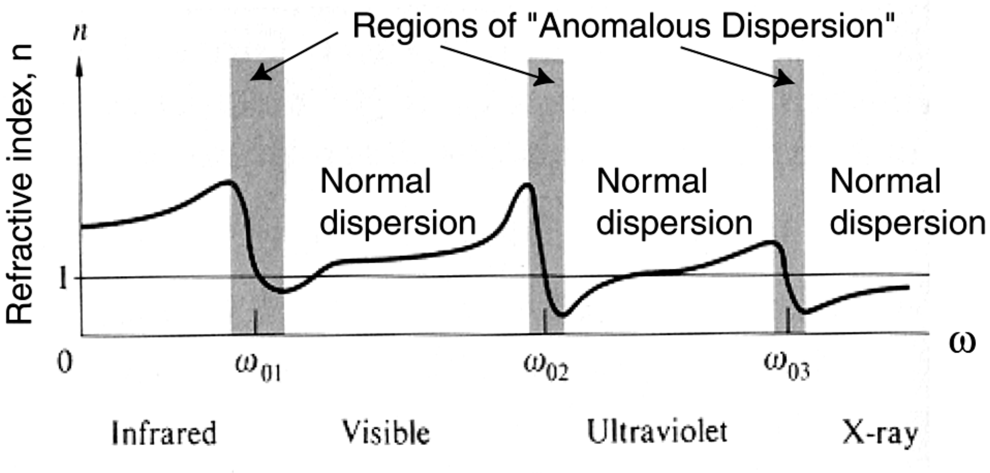
$$v_g = c_0/(n + \omega\, dn/d\omega) = (c_0/n)/(1 + \omega/n\, dn/d\omega)$$

Group velocity = phase velocity (c_0/n), *when* $dn/d\omega = 0$, *such as in vacuum.*

Otherwise, except of regions close to material resonances , n *increases with* ω, $dn/d\omega > 0$, *so*

$$v_g < (c_0/n)$$

Group velocity - normal dispersion regime



Normal material dispersion:
$$\boxed{n(\omega_{blue}) > n(\omega_{yellow}) > n(\omega_{red}) > 1, \quad \frac{dn}{d\omega} > 0}$$

$$v_g = c_0 / (n + \omega\, dn/d\omega)$$

$$\mathbf{v}_g < c$$

$\mathbf{v}_g$ well characterizes velocity of energy carried by a pulse

"Group velocity" in anomalous dispersion regime

$$\boxed{\frac{dn}{d\omega} < 0} \longrightarrow \boxed{V_g = c_o/(n-\omega|dn/d\omega|) > c_o/n}$$

Due to strong pulse distortions **v_g looses the meaning of energy carried by a pulse,** and can even be negative

BUT… signal front velocity never exceeds *c_o* ! (in fact it is = *c_o*)

⟶ **Information cannot be sent faster than *c_o***

Read more about it in http://www.ict.kth.se/courses/IO2655/index.htm?links.html
Ubder: Electromagnetics, Optics

Chromatic and Group Velocity dispersion

$$\varphi(\omega) = k(\omega)\,L$$

To account for dispersion, expand the phase in a Taylor series:

$$k(\omega)L \;=\; k(\omega_0)L \;+\; k'(\omega_0)\left[\omega-\omega_0\right]L \;+\; \tfrac{1}{2}k''(\omega_0)\left[\omega-\omega_0\right]^2 L+\ldots$$

$$k(\omega_0)=\frac{\omega_0}{\mathrm{v}_\phi(\omega_0)} \qquad k'(\omega_0)=\frac{1}{\mathrm{v}_g(\omega_0)} \qquad k''(\omega)=\frac{d}{d\omega}\left[\frac{1}{\mathrm{v}_g}\right]$$

Phase velocity dispersion

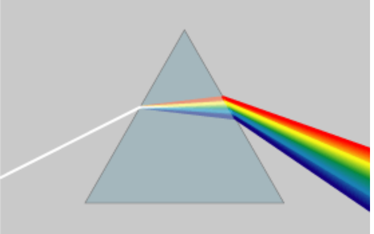
(variation in **phase velocity** with ω, separation of colors in a prism)

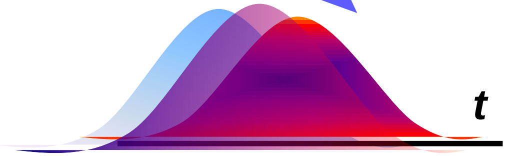
Group velocity dispersion- GVD

(variation in **group velocity** with ω, pulse broadening and "chirp")

|||

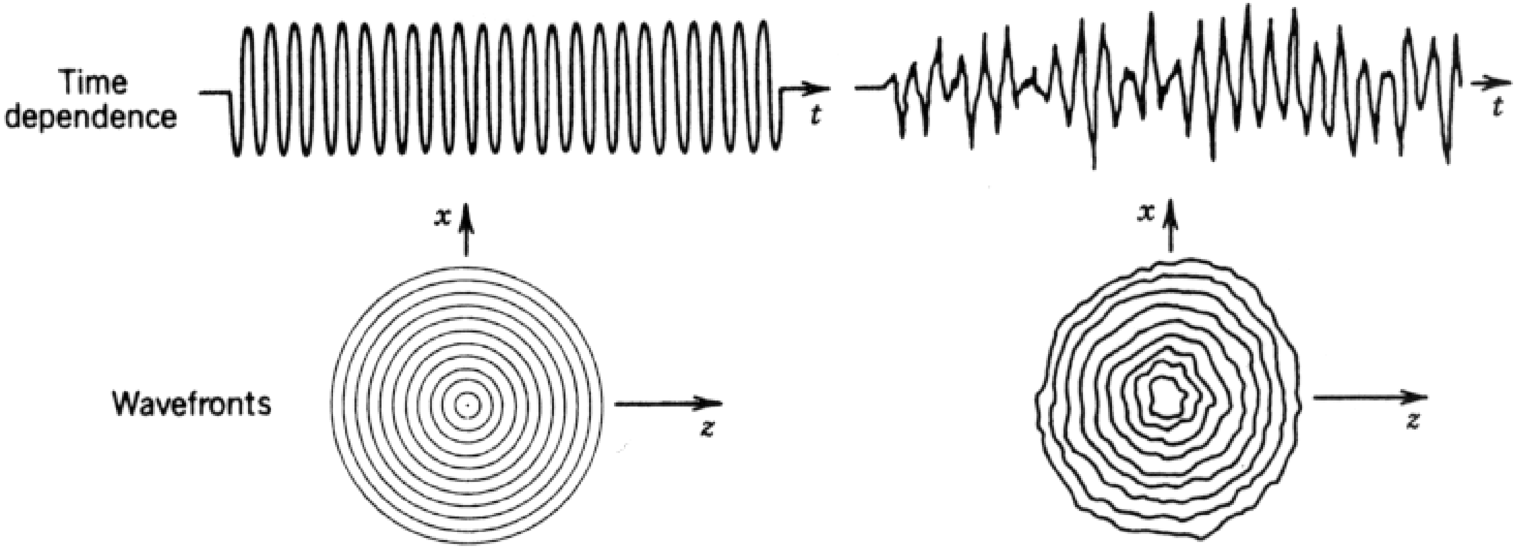
Chromatic dispersion





Coherent and Random light

Temporal cohernce

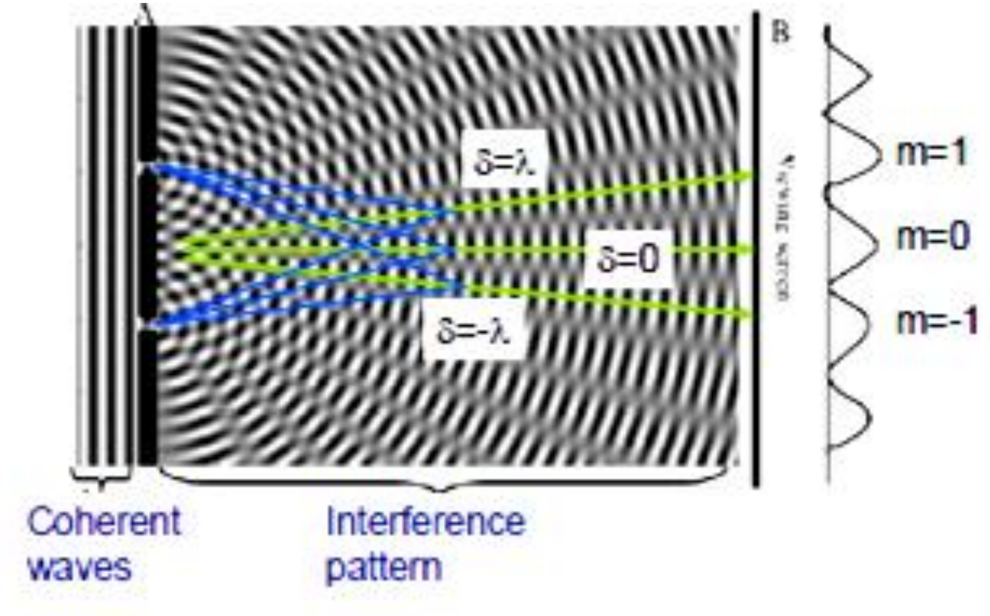


Spatial cohernce

Coherence of waves

Waves are coherent when their relative phase is constant during the resolution time τ_D of the detector - temporal coherence , and within the resolution area A_D - spatial coherence

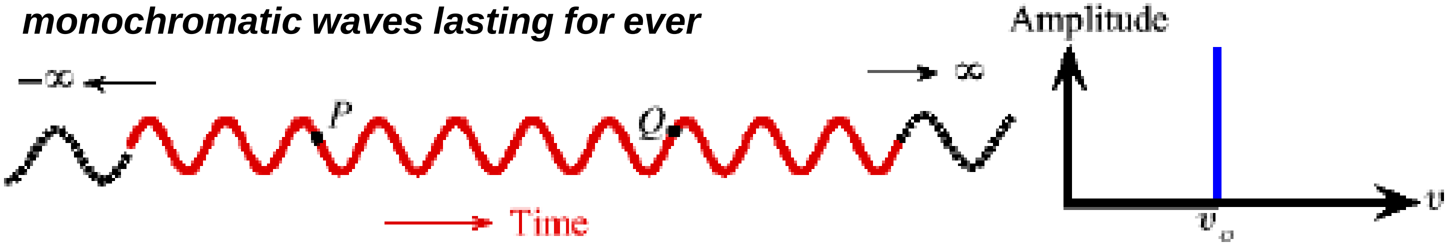
Coherence enables stationary (temporally and spatially constant) interference



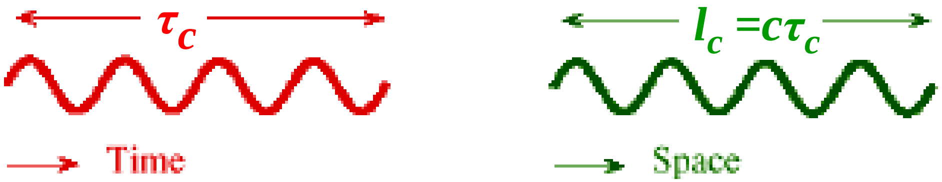
Temporal coherence

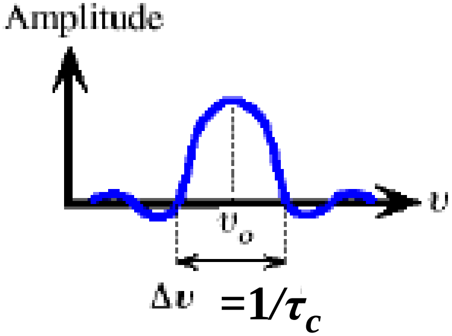
Temporal coherence is a measure of the correlation between the phases of a light wave at different points along the direction of propagation. Temporal coherence tells us how monochromatic a source is.

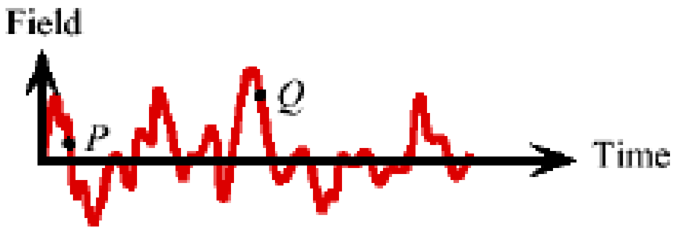
Perfectly coherent – unrealistic would reqire monochromatic waves lasting for ever

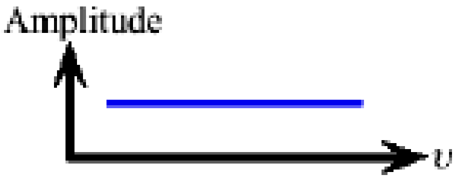


Partially coherent – realistic







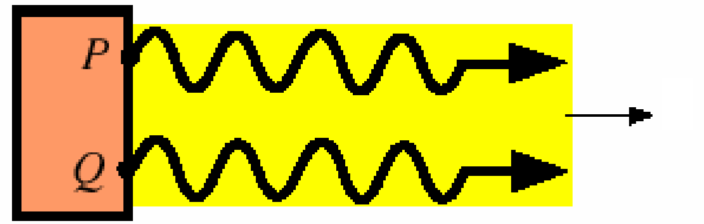


Incoherent – ”white light”

Spatial coherence

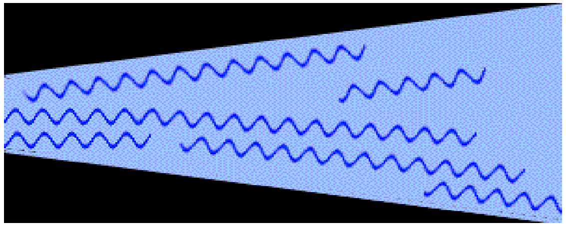
Synchronized phases for rays emitted from different locations on the source <u>during the temporal coherence time</u>. ***Spatial coherence tells us how uniform the phase of the wave front is***

Spacially coherent source



The more extended the source the lower spatial coherence --> Point source would be ideal

Coherence degree - correlation between phase of the wave at two points



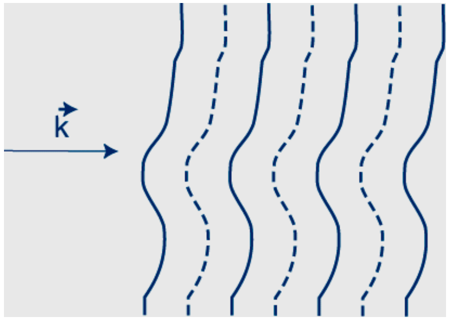
Incoherent beam - large uncertainty in relative phase

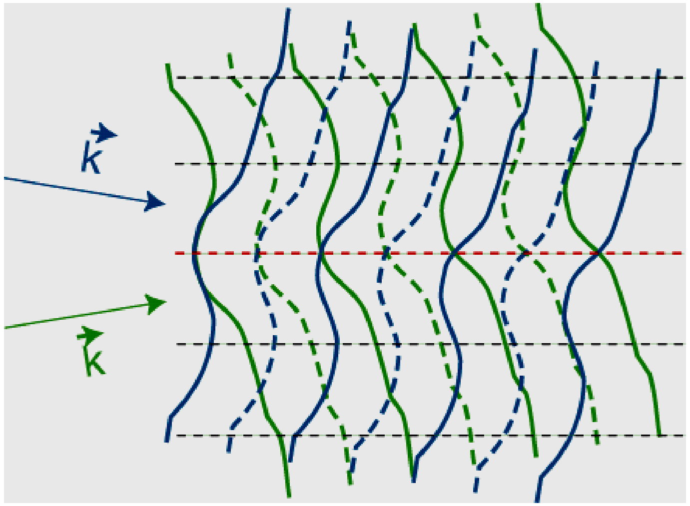
Often, achievable collimation degree is used for assesment of spatial coherence:

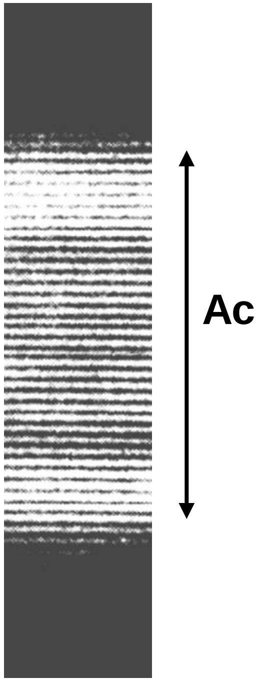
The more collimated the beam the narrower its spectrum in wave vector space (flatter wave front), and the higher spatial coherence

Spatial fringes - coherence area

A beam is temporally but not spatially, coherent:





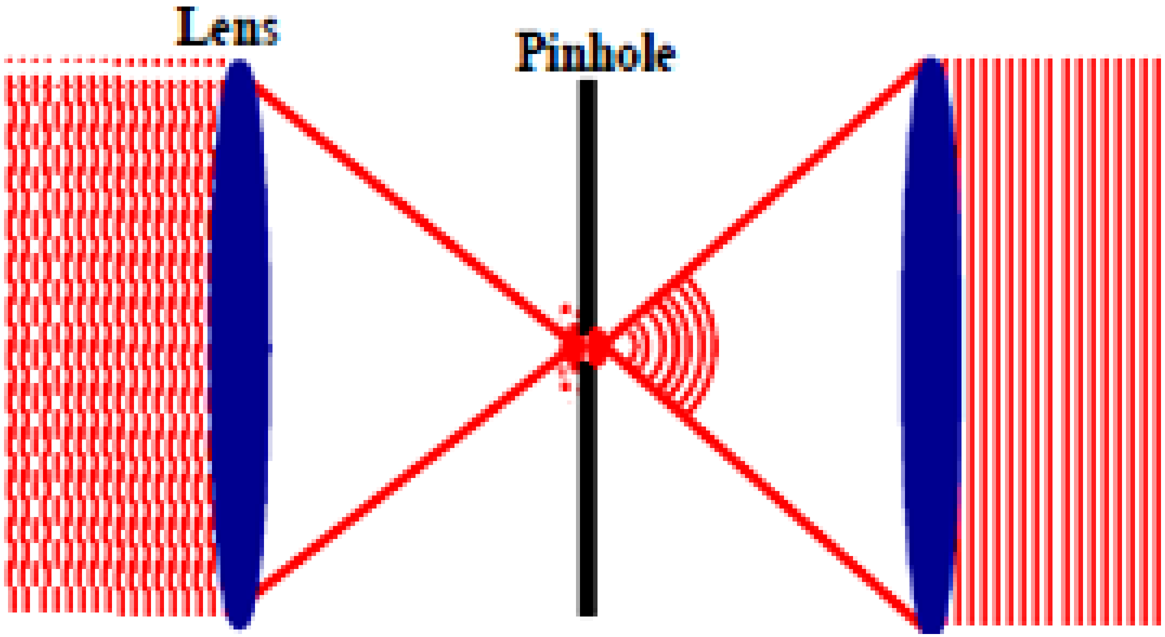


Ac – Coherence area

Interference is coherent (sharp fringes) around the central axis, where same regions of the wave interfere

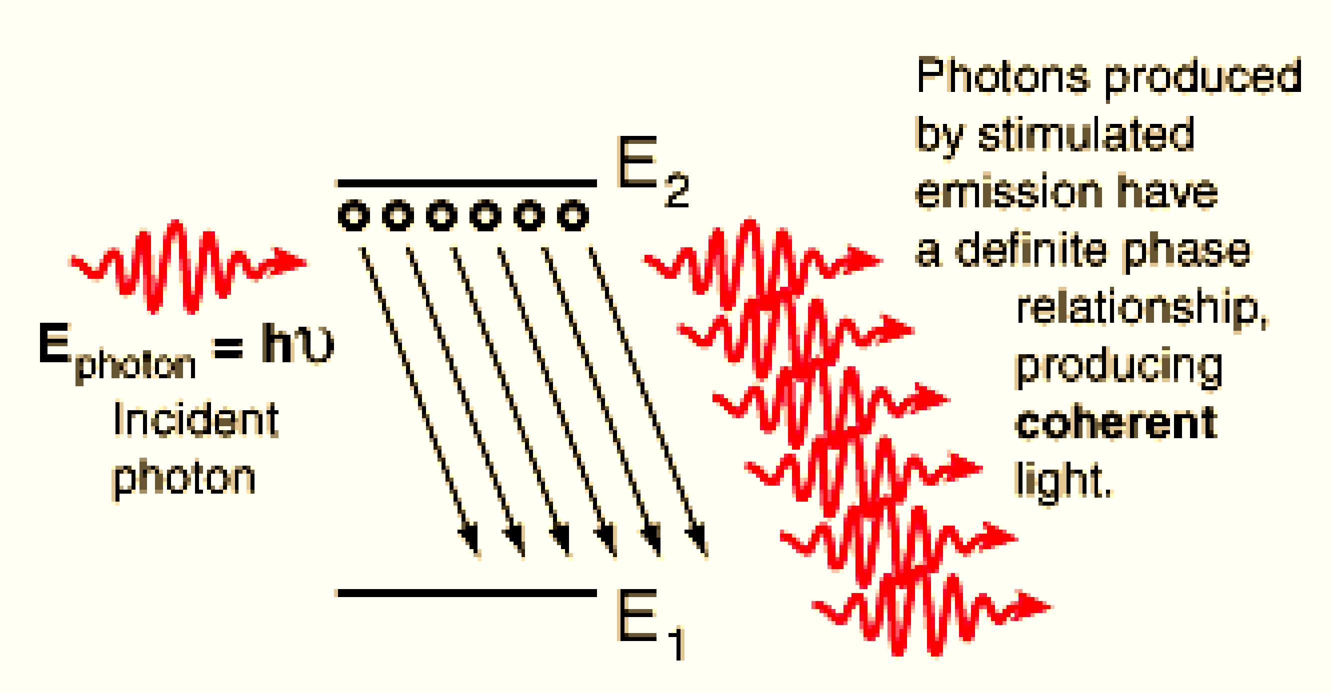
Interference is incoherent (no fringes) far from the axis, where very different regions of the wave interfere

Spatial filtering



The pinhole cleans up spatially incoherent wavefront
It produces a spatially coherent spherical wave (before lens)
or spatially coherent plane wave (after lens)

Laser beam coherence



Photons produced by stimulated emission have a definite phase relationship, producing **coherent** light.

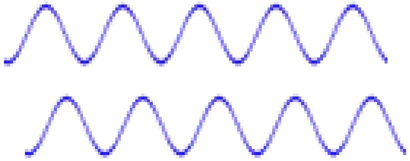
When the laser cavity has flat mirrors the beam is also highly collimated

Mutual (temporal) coherence

When two (or more) waves have the same frequency and their phase difference does not vary in time:

$$\mathbf{E}_1 = \mathbf{A}_1 \exp[i(\omega t - \mathbf{k}_1 \cdot \mathbf{r} + \phi_1)]$$

$$\mathbf{E}_2 = \mathbf{A}_2 \exp[i(\omega t - \mathbf{k}_2 \cdot \mathbf{r} + \phi_2)]$$



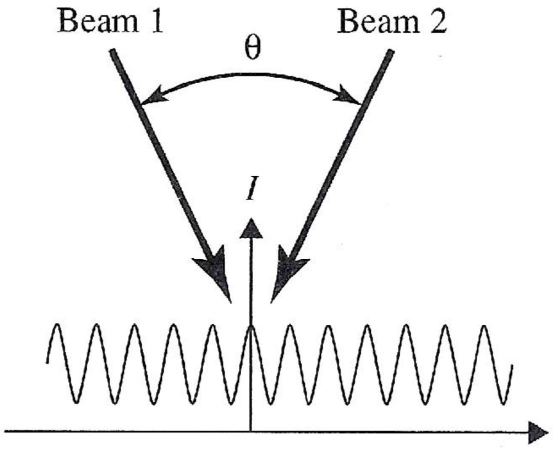
$\Phi_1-\Phi_2=\mathrm{const}$

Their wave vectors must have the same length: $|\mathbf{k}_1| = |\mathbf{k}_2| = \mathbf{k}$, but not direction

Degree of coherence can be characterized by visibility of the interference pattern

$$I = |\mathbf{E}_1 + \mathbf{E}_2|^2 = I_1 + I_2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2 \cos(\mathbf{K} \cdot \mathbf{r} - \phi) = |\mathbf{E}_1 + \mathbf{E}_2|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos(\mathbf{K} \cdot \mathbf{r} - \phi)$$

$$I_i = |\mathbf{E}_i|^2,\ \mathbf{K} = \mathbf{k}_1 - \mathbf{k}_2,\ \text{and}\ \phi = \phi_1 - \phi_2.$$



$$\Lambda = \frac{2\pi}{|\mathbf{K}|} = \frac{\lambda}{2\sin(\theta/2)}$$

Period of the interference fringes

Degree of mutual coherence

$$\gamma_{12} = \left(\frac{\langle\langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle\rangle}{\langle\langle \mathbf{E}_1^* \cdot \mathbf{E}_1 \rangle\rangle^{1/2} \langle\langle \mathbf{E}_2^* \cdot \mathbf{E}_2 \rangle\rangle^{1/2}} \right)_{\mathbf{r}=0} = \langle\langle e^{i(\phi_2-\phi_1)} \rangle\rangle \equiv \frac{1}{\tau_D} \int_0^{\tau_D} e^{i(\phi_2-\phi_1)}\, dt$$

τ_D *- detector time constant* $\gamma_{12} = |\gamma_{12}| \exp(i\alpha)$

If Φ1-Φ2 varies in time the intereference pattern time-averaged by detector *"smears"*

$$\langle\langle I \rangle\rangle = |\mathbf{E}_1 + \mathbf{E}_2|^2 = I_1 + I_2 + 2\mathbf{A}_1 \cdot \underbrace{\mathbf{A}_2 |\gamma_{12}|}_{\text{Modulation depth}} \cos(\mathbf{K} \cdot \mathbf{r} - \alpha)$$

Complete mutual coherence: $|\gamma_{12}| = 1$

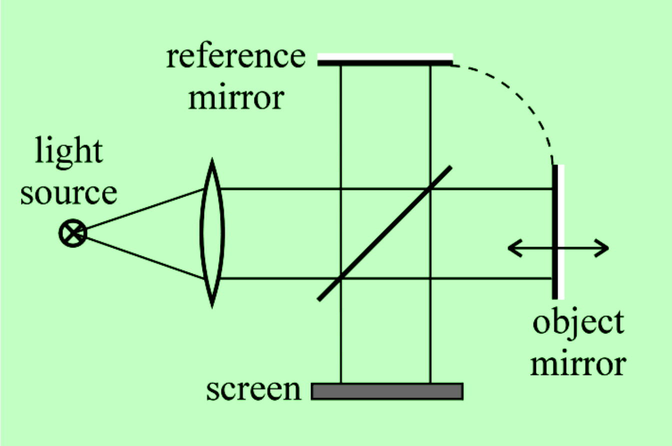
Partial coherence: $0 < |\gamma_{12}| < 1$

Complete mutual incoherence: $|\gamma_{12}| = 0$

Temporal self-coherence

Coherence of two parts of the same wave

Temporal coherence can be measured in a Michelson interferometer
The wave is combined with a copy of itself that is delayed by time τ by moving the object mirror



Coherence time: $\tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau$

The longest time delay for which the phases are correlated (fringes are visible) $\tau_c \to O(1/\Delta\nu)$.
A rule of thumb is that
Coherence length: $l_c = c\tau_c$ *(more practical in the lab)*

The longest propagation length over which coherence is preserved

For LED lc is of the order of microns, for a laser diode – centimeters, for a gas laser – meters!

Degree of temporal self-coherence – formulae

Similar to those for mutual coherence, but here time variation of both intensity and the phase are included by introducing time dependent complex amplitude A(t), so that one can also analyze pulses

Delayed parts of the same beam:

$$E_1 = A(t)\exp[i(\omega t - \mathbf{k}_1 \cdot \mathbf{r})]$$

$$E_2 = A(t+\tau)\exp[i(\omega t + \omega\tau - \mathbf{k}_2 \cdot \mathbf{r})]$$

A(t) – slowly varying amplitude, e.g. envelope of a pulse at the centrall frequency ω

$$\gamma(\tau) = \frac{\langle\langle E^*(t)\, E(t+\tau)\rangle\rangle}{\langle\langle E^*(t)\, E(t)\rangle\rangle^{1/2}\langle\langle E^*(t+\tau)\, E(t+\tau)\rangle\rangle^{1/2}}$$

$$\gamma(\tau) = \frac{\langle\langle E^*(t)\, E(t+\tau)\rangle\rangle}{\langle\langle E^*(t)\, E(t)\rangle\rangle}$$

Average correlation between field value at any pair of times, separated by delay τ