SOLVABILITY OF GENERALIZED HAMILTONIAN SYSTEMS I : INTRODUCTION

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We regard a subset S of the tangent bundle TM of a manifold M as an ordinary differential equation and call it an implicit differential equation. A solution of an implicit differential equation S is a smooth curve c(t) of M such that (c(t), dc/dt(t)) are contained in S for all t. In the three lectures, we consider a special type of implicit differential equations called Generalized Hamiltonian Systems introduced to Mathematics by P.A.M. Dirac. We regard a Lagrangian submanifold L of the tangent space TM of a symplectic manifold M as an implicit differential equation. A Lagrangian submanifold L is called a Generalized Hamiltonian System if it is a fiber bundle over a submanifold K of M. Then its generating family is linear with respect to parameters. To the generating family, we associate a linear equation smoothly depending on points of M and parameters. We show that solvability of the generalized Hamiltonian system is related to existence of a smooth solution of the linear equation. In this introductory talk we observe the simplest case where the rank of the matrix of the linear equation is constant.

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