Lecture III

Surfaces in 4-spaces

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In this lecture we study the geometry of surfaces in 4-space based on the analysis of their contacts with k-planes and k-spheres.

First we review the classical results on the second order geometry of surfaces, based on the remarkable work of J. Little, 1969. The second fundamental form of the surface is given by two quadratic forms. The geometry of this form is described in terms of a classical object: *the curvature ellipse.*

In recent years, the subject was pushed forward by the work of many people, including J. Montaldi, D. Mond, M.C.Romero-Fuster, M.Ruas, D. Mochida, J.W.Bruce, F. Tari, J. J. Nunno-Ballesteros and D. Dreibelbis. We review some of these results, emphasizing the geometric methods in the investigation of the higher codimensional situation.

The following topics will be discussed:
- Classical results on the second order geometry of surfaces in 4-space;
- The flat geometry of surfaces in 4-space;
- The differential equation of the asymptotic lines;
- Global results.